

Test Paper Code : PH

Time : 3 Hours Maximum Marks : 300

### INSTRUCTIONS

1. The question-cum-answer book has 36 pages and has 25 questions. Please ensure that the copy of the question-cum-answer book you have received contains all the questions.
2. Write your **Roll Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.
3. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on Page No. 11. Do not write anything else on this page.
4. Each objective question has 4 choices for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used :
  - (a) For each objective question, you will be awarded **6 (six)** marks if you have written only the correct answer.
  - (b) In case you have not written any answer for a question, you will be awarded **0 (zero)** mark for that question.
  - (c) In all other cases, you will be awarded **-2 (minus two)** marks for the question.
  - (d) Negative marks for objective part will be carried over to total marks.
5. Answer the subjective question only in the space provided after each question.
6. Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing later only will be evaluated.
7. All answers must be written in blue/black/blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
8. All rough work should be done in the space provided and scored out finally.
9. No supplementary sheets will be provided to the candidates.
10. **Logarithmic Tables / Calculator of any kind / cellular phone / pager / electronic gadgets are not allowed.**
11. The question-cum-answer book must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this book.

READ THE INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

| ROLL NUMBER   |  |  |  |  |
|---------------|--|--|--|--|
|               |  |  |  |  |
| Name :        |  |  |  |  |
| Test Centre : |  |  |  |  |

**Do not write your Roll Number or Name anywhere else in this question-cum-answer book.**

I have read all the instructions and shall abide by them.

.....  
Signature of the Candidate

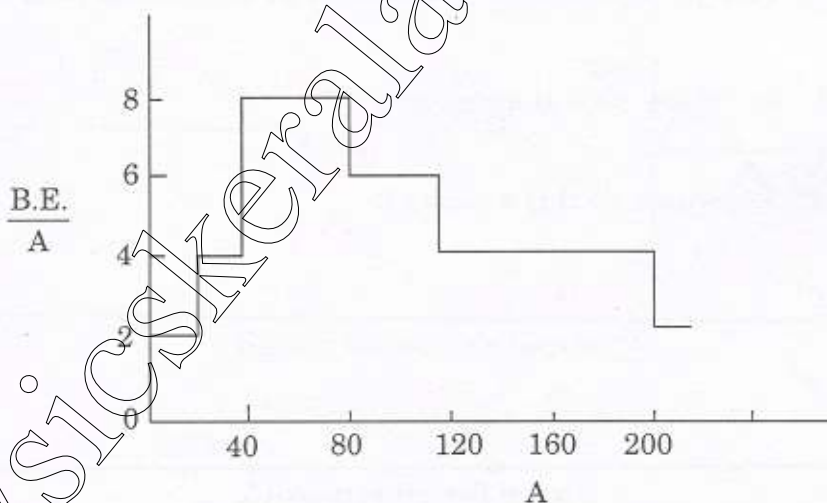
I have verified the information filled by the Candidate above.

.....  
Signature of the Invigilator

**IMPORTANT NOTE FOR CANDIDATES**

- Attempt ALL the 25 questions.
- Questions 1-15 (objective questions) carry six marks each and questions 16-25 (subjective questions) carry twenty one marks each.
- Write the answers to the objective questions in the Answer Table for Objective Questions provided on page 11 only.

1. Fermi energy of a certain metal  $M_1$  is 5 eV. A second metal  $M_2$  has an electron density which is 6% higher than that of  $M_1$ . Assuming that the free electron theory is valid for both the metals, the Fermi energy of  $M_2$  is closest to
- (A) 5.6 eV  
 (B) 5.2 eV  
 (C) 4.8 eV  
 (D) 4.4 eV
2. The following histogram represents the binding energy per particle (B.E./A) in MeV as a function of the mass number A of a nucleus.

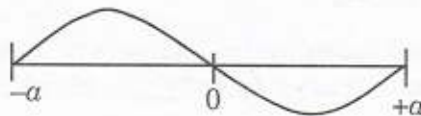


A nucleus with mass number  $A = 180$  fissions into two nuclei of equal masses. In the process

- (A) 180 MeV of energy is released  
 (B) 180 MeV of energy is absorbed  
 (C) 360 MeV of energy is released  
 (D) 360 MeV of energy is absorbed



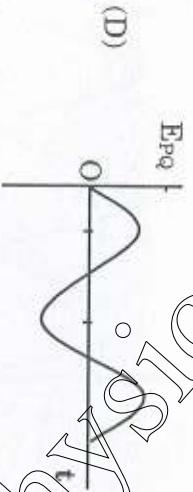
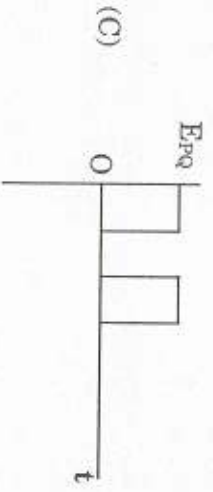
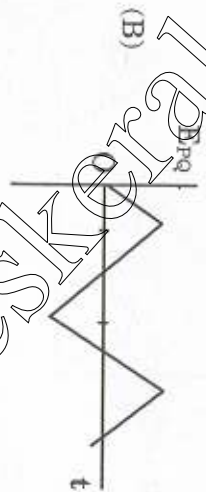
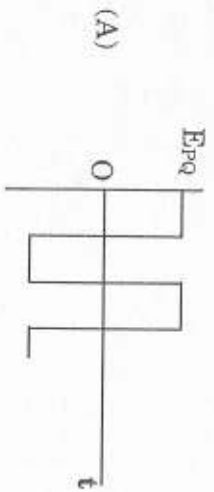
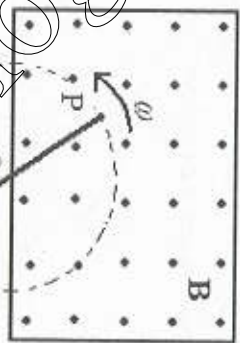
3. A particle is confined in a one dimensional box with impenetrable walls at  $x = \pm a$ . Its energy eigenvalue is 2 eV and the corresponding eigenfunction is as shown below.



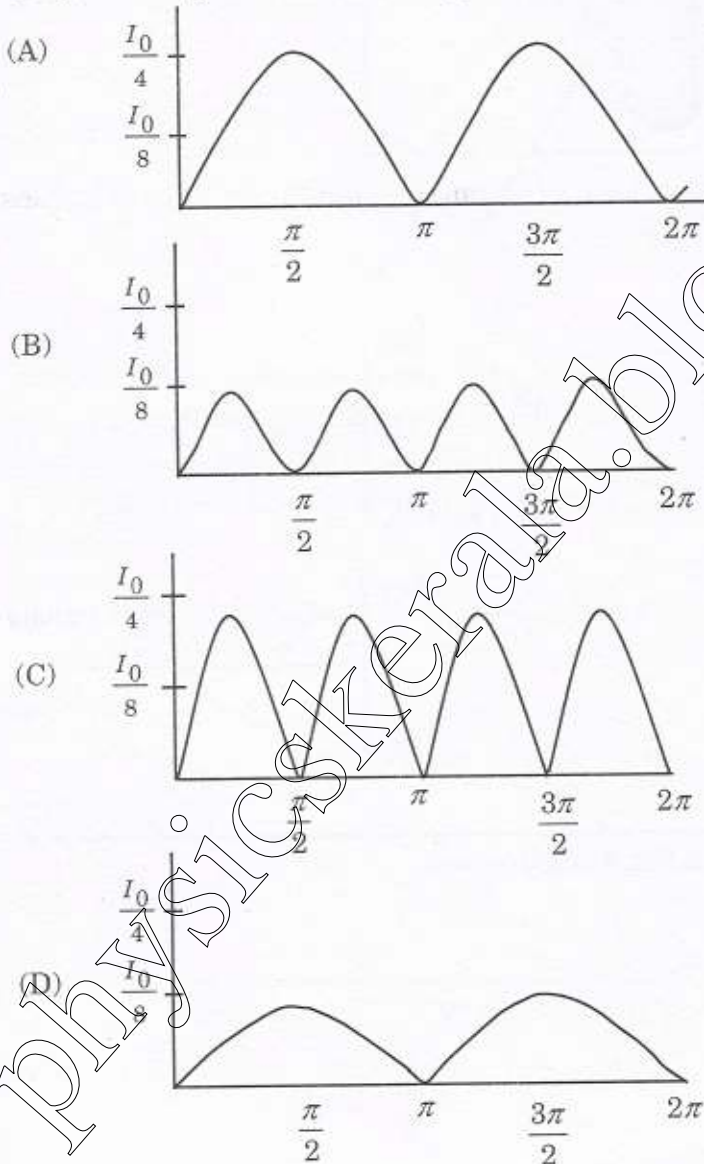
The lowest possible energy of the particle is

- (A) 4 eV  
 (B) 2 eV  
 (C) 1 eV  
 (D) 0.5 eV
4. Experimental measurements of heat capacity per mole of Aluminium at low temperatures show that the data can be fitted to the formula,  $C_V = aT + bT^3$ , where  $a = 0.00135 \text{ J K}^{-2} \text{ mole}^{-1}$ ,  $b = 2.48 \times 10^{-5} \text{ J K}^{-4} \text{ mole}^{-1}$  and  $T$  is the temperature in Kelvin. The entropy of a mole of Aluminium at such temperatures is given by the formula
- (A)  $aT + \frac{b}{3}T^3 + c$ , where  $c > 0$  is a constant  
 (B)  $\frac{aT}{2} + \frac{b}{4}T^3 + c$ , where  $c > 0$  is a constant  
 (C)  $aT + \frac{b}{3}T^3$   
 (D)  $\frac{aT}{2} + \frac{b}{4}T^3$

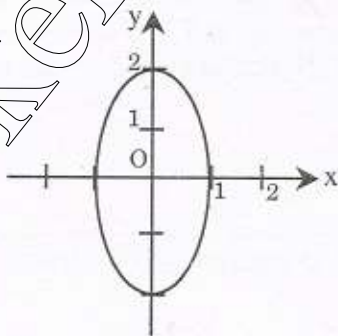
5. A uniform and constant magnetic field  $B$  coming out of the plane of the paper exists in a rectangular region as shown in the figure. A conducting rod  $PQ$  is rotated about  $O$  with a uniform angular speed  $\omega$  in the plane of the paper. The emf  $E_{PQ}$  induced between  $P$  and  $Q$  is best represented by the graph



6. Three polarizers P, Q and R are placed parallel to each other with their planes perpendicular to the z-axis. Q is placed between P and R. Initially the polarizing directions of P and Q are parallel, but that of R is perpendicular to them. In this arrangement when unpolarized light of intensity  $I_0$  is incident on P, the intensity coming out of R is zero. The polarizer Q is now rotated about the z-axis. As a function of angle of rotation, the intensity of light coming out of R is best represented by



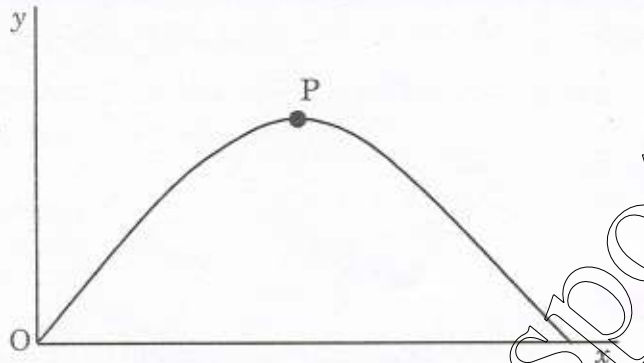
7. The black body spectrum of an object  $O_1$  is such that its radiant intensity (i.e., intensity per unit wavelength interval) is maximum at a wavelength of 200 nm. Another object  $O_2$  has the maximum radiant intensity at 600 nm. The ratio of power emitted per unit area by  $O_1$  to that of  $O_2$  is
- (A)  $\frac{1}{81}$   
 (B)  $\frac{1}{9}$   
 (C) 9  
 (D) 81
8. In terms of the basic units of mass (M), length (L), time (T) and charge (Q), the dimensions of magnetic permeability of vacuum ( $\mu_0$ ) are
- (A)  $MLQ^{-2}$   
 (B)  $ML^2T^{-1}Q^{-2}$   
 (C)  $LTQ^{-1}$   
 (D)  $LT^{-1}Q^{-1}$
9. When two simple harmonic oscillations represented by  $x = A_0 \cos(\omega t + \alpha)$  and  $y = B_0 \cos(\omega t + \beta)$  are superposed at right angles, the resultant is an ellipse with its major axis along the y-axis as shown in the figure. The conditions which correspond to this are



- (A)  $\beta = \alpha + \frac{\pi}{2}; A_0 = 2B_0$   
 (B)  $\beta = \alpha - \frac{\pi}{4}; A_0 = B_0$   
 (C)  $\beta = \alpha + \frac{\pi}{2}; 2A_0 = B_0$   
 (D)  $\beta = \alpha + \frac{\pi}{4}; A_0 = B_0$



10. A projectile is fired from the origin O at an angle of  $45^\circ$  from the horizontal. At the highest point P of its trajectory the radial and transverse components of its acceleration in terms of the gravitational acceleration  $g$  are



- (A)  $a_r = \frac{2g}{\sqrt{5}}$ ,  $a_\theta = \frac{g}{\sqrt{5}}$
- (B)  $a_r = \frac{-2g}{\sqrt{5}}$ ,  $a_\theta = \frac{-g}{\sqrt{5}}$
- (C)  $a_r = \frac{g}{\sqrt{5}}$ ,  $a_\theta = \frac{2g}{\sqrt{5}}$
- (D)  $a_r = \frac{-g}{\sqrt{5}}$ ,  $a_\theta = \frac{-2g}{\sqrt{5}}$
11. A satellite moves around a planet in a circular orbit at a distance  $R$  from its centre. The time period of revolution of the satellite is  $T$ . If the same satellite is taken to an orbit of radius  $4R$  around the same planet, the time period would be
- (A)  $8T$
- (B)  $4T$
- (C)  $T/4$
- (D)  $T/8$
12. The speed of an electron, whose de Broglie wavelength is equal to its Compton wavelength, is ( $c$  is the speed of light)
- (A)  $c$
- (B)  $c/\sqrt{2}$
- (C)  $c/2$
- (D)  $c/3$

13.  $(x \ y) \begin{pmatrix} 5 & -7 \\ 7 & 3 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = 15$

The matrix equation above represents

- (A) a circle of radius  $\sqrt{15}$
- (B) an ellipse of semi major axis  $\sqrt{5}$
- (C) an ellipse of semi major axis 5
- (D) a hyperbola

14.

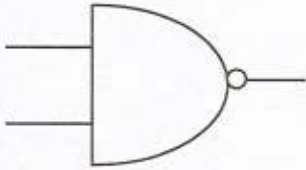


Fig (i)

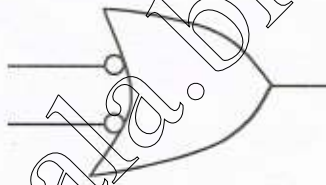


Fig (ii)

Figures (i) and (ii) represent respectively,

- (A) NOR, NOR
- (B) NOR, NAND
- (C) NAND, NAND
- (D) OR, NAND

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Space for rough work



15.  $f(x)$  is a periodic function of  $x$  with a period of  $2\pi$ . In the interval  $-\pi < x < \pi$ ,  $f(x)$  is given by

$$f(x) = \begin{cases} 0 & -\pi < x < 0 \\ \sin x & 0 < x < \pi \end{cases}$$

In the expansion of  $f(x)$  as a Fourier series of sine and cosine functions, the coefficient of  $\cos(2x)$  is

- (A)  $\frac{2}{3\pi}$
- (B)  $\frac{1}{\pi}$
- (C) 0
- (D)  $-\frac{2}{3\pi}$

**Answer Table for Objective Questions**

Write the Code of your chosen answer only in the 'Answer' column against each Question No. Do not write anything else on this page.

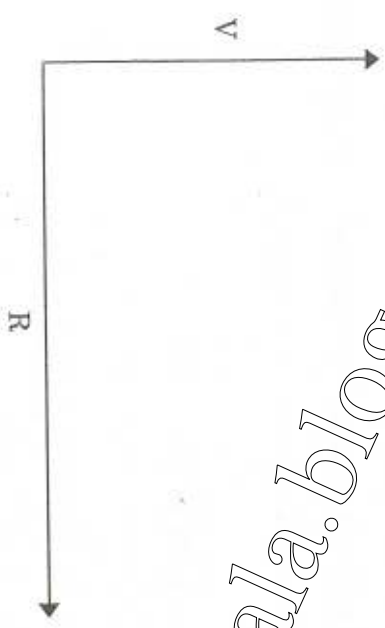
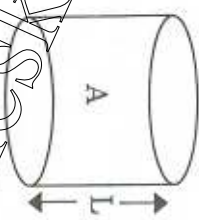
| Question No. | Answer | Do not write in this column |
|--------------|--------|-----------------------------|
| 01           |        |                             |
| 02           |        |                             |
| 03           |        |                             |
| 04           |        |                             |
| 05           |        |                             |
| 06           |        |                             |
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| 10           |        |                             |
| 11           |        |                             |
| 12           |        |                             |
| 13           |        |                             |
| 14           |        |                             |
| 15           |        |                             |

**FOR EVALUATION ONLY**

|                                   |  |       |       |
|-----------------------------------|--|-------|-------|
| No. of Correct Answers            |  | Marks | ( + ) |
| No. of Incorrect Answers          |  | Marks | ( - ) |
| Total Marks in Question Nos. 1-15 |  |       | ( )   |

16. If the total surface area (including the area of the top and bottom ends) of a cylinder is to be kept fixed ( $=A$ ), what is its maximum possible volume?

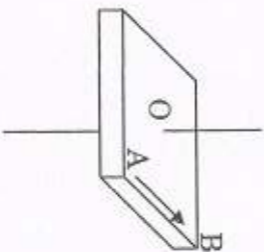
For such cylinders of fixed total area, plot in the axes shown below their volume ( $V$ ) versus the radius ( $R$ ) clearly indicating the values of  $R$  for which the volume is maximum and zero.



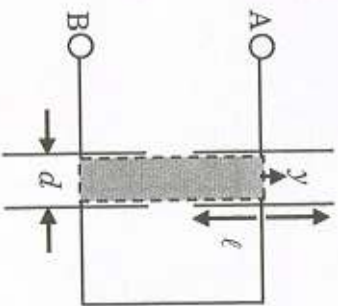


17.

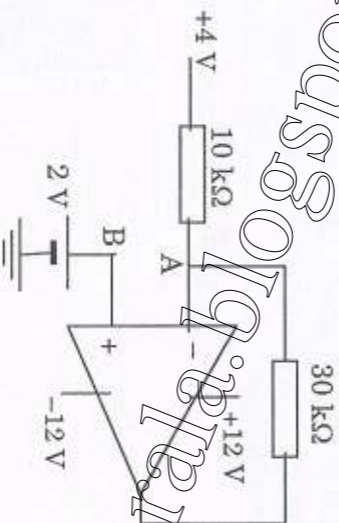
A horizontal square platform of mass  $m$  and side  $a$  is free to rotate about a vertical axis passing through its centre  $O$ . The platform is stationary and a person of the same mass ( $m$ ) as the platform is standing on it at point  $A$ . The person now starts walking along the edge from  $A$  to  $B$  (see figure). The speed  $v$  of the person with respect to the platform is constant. Find the time the person takes to reach  $B$ . Also find his distance  $r(t)$  from  $O$  as a function of time. Further find the angle through which the platform has rotated by the time the person reaches  $B$ .



18. Two identical parallel plate capacitors are connected across terminals A and B as shown. Each of the capacitors is made of square plates of side  $\ell$  with a distance  $d$  between them. A dielectric slab (relative permittivity  $k$ ) of thickness  $d$  is kept between the plates. The slab covers only half of the length of the plates in each of the capacitors as shown. Find the total capacitance of the assembly. The capacitors are charged by a battery and then the battery is disconnected. If the slab is now displaced slightly by a distance  $y$  [ $(y/\ell) \ll 1$ ], show that it will perform simple harmonic oscillations.



19. For the circuit shown below, calculate the output voltage  $V_o$ . What would  $V_o$  be if the polarity of 2 V battery is reversed at terminal B? (Assume the operational amplifier to be ideal).

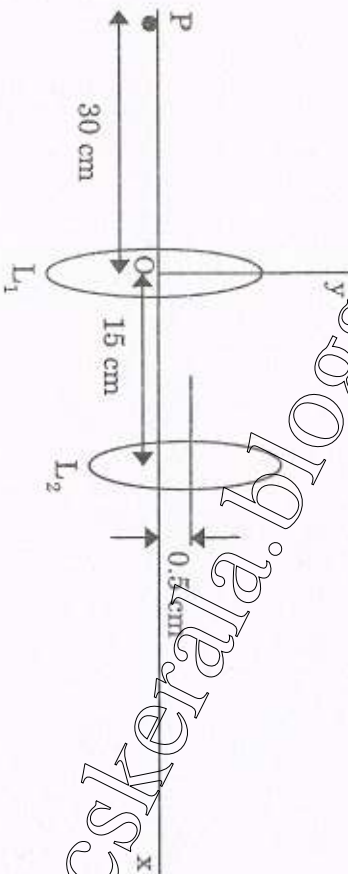




20. A beam of light of wavelength 400 nm and power 1.55 mW is directed at the cathode of a photoelectric cell (work function = 2.40 eV). If only 10% of the incident photons effectively produce photoelectrons, find the current due to these electrons. If the wavelength of light is now reduced to 200 nm keeping its power the same, the kinetic energy of the electrons is found to increase by a factor of 5. What are the values of the stopping potentials for the two wavelengths?

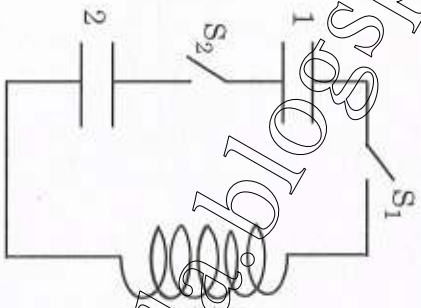
21.

Two thin lenses  $L_1$  and  $L_2$  of focal lengths 15 cm and 10 cm respectively, are kept 15 cm apart from each other. Their axes are separated by 0.5 cm as shown in figure (not to scale). If a point object P is placed on the axis of  $L_1$  to its left at a distance of 30 cm, find the x and y coordinates (origin O) of the image formed by the combination.



22.

The circuit shown consists of two identical capacitors of capacitance  $C$  each and an inductor of inductance  $L$ . Initially, both switches are open and capacitor 1 is charged with a charge  $Q_0$ . When the second capacitor has no charge. Switches  $S_1$  and  $S_2$  are closed simultaneously at  $t = 0$ . The circuit now becomes oscillatory.



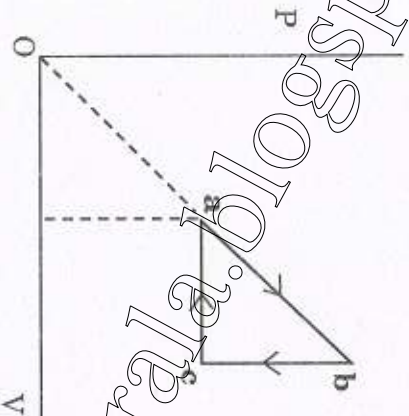
(a) Calculate the maximum current in the circuit. [12]

(b) Obtain expressions for the charge on the capacitors 1 and 2 as a function of time. [9]



23. A particle travels along the diameter of the earth at a relativistic speed. It crosses the earth in a time  $3 \times 10^{-7}$  s in its own frame. An observer, located on the earth, measures the same time interval to be  $5 \times 10^{-2}$  s. Find the speed of the particle with respect to the earth and the diameter of the earth.

24.  $1 \text{ m}^3$  of an ideal gas with  $\gamma = C_P/C_V = 1.5$  is at a pressure of  $100 \text{ kPa}$  and a temperature of  $300 \text{ K}$ . Initially the state of the gas is at the point **a** of the  $PV$  diagram shown. The gas is taken through a reversible cycle  $a \rightarrow b \rightarrow c \rightarrow a$ . The pressure at point **b** is  $200 \text{ kPa}$  and the line  $ba$ , when extended, passes through the origin.



- (a) Calculate the work done by the gas in each of the steps  $a \rightarrow b$ ,  $b \rightarrow c$  and  $c \rightarrow a$ . [9]
- (b) Calculate the change in entropy of the gas in each of the three steps above. [12]

25. How much work is done when an object moves from  $O \rightarrow P \rightarrow Q \rightarrow R \rightarrow O$  in a force field given by

$$F(x, y) = (x^2 - y^2) \hat{i} + 2xy \hat{j}$$

along the rectangular path shown. Find the answer by evaluating the line integral and also by using the Stokes' theorem.

